UNIVERSITÉ DE MONTRÉAL

**CENTRE DE RECHERCHE SUR LES TRANSPORTS**

CASE POSTALE 6128, SUCCURSALE A, MONTRÉAL, QUÉBEC, H3C 3J7

# A LOCATION PROBLEM WITH MINIMAX OBJECTIVE

by

Horst A. Eiselt*

* Concordia University
7141 Sherbrooke Street West, Montreal, Quebec H4B 1R6

Université de Montréal

Centre de recherche sur les transports - Publication #432
École des Hautes Etudes Commerciales - Cahiers du GERAD G-85-13
Septembre 1985

Dépôt légal - 4e trimestre 1985

# A LOCATION PROBLEM WITH MINIMAX OBJECTIVE*

by

Horst A. Eiselt
Concordia University
7141 Sherbrooke Street West
Montréal H4B 1R6
Canada

Résumé: Dans cet article, on décrit un problème de localisation de type minimax dans le plan: l'objectif consiste à minimiser le flux maximal en un point quelconque du plan. On développe des méthodes de solution pour les versions continue et discrète du problème et on présente des résultats numériques. On discute également certaines extensions du problème.

---

* This paper was written while the author was on sabbatical leave at the Ecole des Hautes Etudes Commerciales de Montréal in Winter 1985.

# A LOCATION PROBLEM WITH MINIMAX OBJECTIVE

H.A. Eiselt, Concordia University, Montreal, Canada

**Abstract**

In this paper a minimax location problem in the plane is described where it is the objective to minimize the largest flow at any point in the plane. Solution techniques for a continuous and a discrete version of the problem are developed and computational experience is reported. Some extensions of the problems are also discussed.

## 1. Introduction

Let n points $P_j$ with coordinates $(x_j, y_j)$, $j = 1, \ldots, n$ be given in the plane. To each of these points a positive number $s_j \in \mathbb{R}_+$ is assigned. In the following these numbers are referred to as supplies. Whenever supply units are shipped from any one point to another these shipments will be restricted to

- moves parallel to the axes
- moves with no more than a single turn.

The latter assumption guarantees that the length of the selected path is the shortest possible one. Reasons for these restrictions are explained below. It is now the goal to find a point $P^* = (x^*, y^*)$ so that the flow which is generated by shipping the supplies of all points $P_j$ to $P^*$ satisfies certain criteria. These criteria are discussed after describing some possible applications of the above problem. In principle, this problem is of the location-allocation type where the first task is to locate a point $P^*$ and then an allocation (or routing or traffic assignment) procedure determines the flow pattern.

Some applications of this problem are as follows. Let the area under consideration be the core of some city where the given points

represent large office buildings, shopping malls etc. The supplies of these points indicate the approximate number of individuals being located at the respective points at any given time. It is now the goal to plan the location of an emergency shelter, such that moving all individuals from their current locations to the shelter causes minimal panic. Clearly, surrogate criteria will have to be found in order to model "panic". Some of those criteria are introduced and discussed below. For a further discussion of evacuation models the reader is referred to /SMP 82/. Another situation in which only the "allocation" part is relevant can be described as follows. Let the given points represent highway exits or other places on the outskirts of town where traffic into town accumulates. Moreover, the point P* denotes a "place of attraction", such as the parking lot next to a ballpark. The problem is to find a traffic assignment that minimizes traffic congestion or any chaos in general. Other applications of this problem can be found in the area of military logistics.

## 2. Minimax Location Problems in the Plane

It proves useful for the following analyses to cover all given points by horizontal as well as vertical lines. As a result, every point in the plane (and particularly P*) is located in one of these positions:

(1) at one of the n given supply points

(2) at one of the $O(n^2)$ non-supply points, generated by intersecting horizontal and vertical lines

(3) on one of the $O(n^2)$ open line segments between adjacent supply and/or non-supply points

or (4) in one of the $O(n^2)$ open areas bounded by line segments and supply and/or non-supply points.

For the time being, it is assumed that P* has been tentatively located. This location will be called an experiment. If P* is at its optimal location, it is called a minimax center or MMC for short. For a discussion of MMC's in some special graphs, see /EP 84/. If one considers P* as the origin of a system of coordinates, then every point in the plane is either located in one of the quadrants of on one of the four axes. Let $s_I(P^*)$, $s_{II}(P^*)$, $s_{III}(P^*)$ and $s_{IV}(P^*)$ denote the sum of the

supplies of all points, located in the respective quadrant, generated by P*. For notational convenience we will simply use $s_I$, $s_{II}$, $s_{III}$ and $s_{IV}$ if no confusion can arise. Similarly, let $f_N$, $f_W$, $f_S$ and $f_E$ denote the sum of the supplies on the respective axes north, west, south and east of P*. Defining $s := s_I + s_{II} + s_{III} + s_{IV}$ and $f := f_N + f_W + f_S + f_E$, the total supply is s + f (plus the supply at P* if it is a supply point). Now the flow from a given point $P_j$ in, say, the first quadrant to P* is as follows. Any unit in $P_j$ is either first shipped to the northern axis and it then approaches P* from the north or it is first sent to the eastern axis and then westward to P*. On the other hand, all units on any of the given axes can only be shipped on the corresponding straight line to P*. This peculiar flow pattern has the following property.

Lemma 1: Flows accumulate towards P*.

In other words, if flows from different points are joined somewhere, then they are not separated again. This also leads to the fact that on any of the paths used for the shipments from some point $P_j$ to P* the number of flow units increases monotonically towards P* with a single exception, viz. if two paths cross. This case is further elaborated upon below.

In order to formulate the problem, a surrogate criterion for "minimize chaos" will have to be found. Here it is assumed that the likelihood for a panic or chaos is increasing whenever the flow of individuals increases. Thus the objective could be to minimize the maximal flow at any point in the plane. Since flows can approach P* only from the four directions north, west, south and east and lemma 1 holds, therefore the largest flow at any one point in the plane (if crossings can be avoided) will occur directly adjacent to P*. Define $x_j$ as the proportion of the supply at the point $P_j$ which is first shipped north/south to the horizontal axis and from there to P*. Moreover, let I (II, III, IV) be the set of indices of all points in the first (second, third, fourth) quadrant. Points on the axes are not assigned a variable since their flow to P* is unique and fixed on the corresponding axis. Now the flow on the western axis into P* is $W = f_W + \sum_{j \in II \cup III} s_j x_j$ and

the flow from the north into P* is $N = f_N + s_I + s_{II} - \sum_{j \in I \cup II} s_j x_j$ and similarly for S and E. Then the continuous individual flow problem can be written as

$$P_{CI}: \quad \text{Min } z = \max \{(f_N + s_I + s_{II} - \sum_{j \in I \cup II} s_j x_j), (f_W + \sum_{j \in II \cup III} s_j x_j),$$

$$(f_S + s_{III} + s_{IV} - \sum_{j \in III \cup IV} s_j x_j), (f_E + \sum_{j \in IV \cup I} s_j x_j)\}$$

$$\text{s.t.} \quad x_j \in [0; 1] \; \forall \; j$$

Before solving the allocation problem $P_{CI}$ above we will briefly mention some parameters other than the magnitude of the flow where chaos can occur.

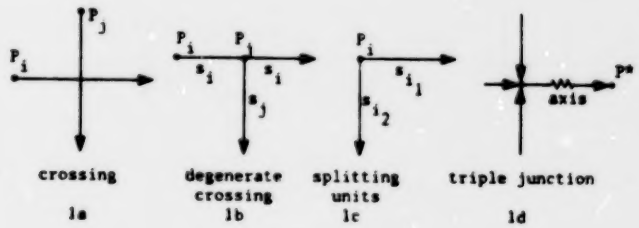


Figure 1

One of these configurations which is most detrimental to a steady and efficient flow is a crossing as shown in figure 1a. Even if simple and unequivocal rules are given for routing the flows in a case like this (such as traffic lights, preference rules) congestion will occur. Note that in our problem it is not possible to route part of the flow originating at $P_j$ first south and then east, the remaining part due south and the whole supply originating at $P_i$ first east and then south, thus avoiding a crossing. Such a flow pattern would not satisfy the single turn rule as outlined in the beginning of this paper. The situation displayed in figure 1b is essentially the same as the one in figure 1a with the point $P_j$ moved downwards, thus the name "degenerate crossing". Another problem may occur when splitting units as shown in figure 1c. Confusion may arise at this point since some units are routed south and some east. Specifically in evacuation-type situations, simplicity of the rules will have to be one of the major considerations. Also, one should make an effort to avoid triple junctions such as the one shown in figure 1d. The problems with flows from up to three different

directions meeting at a single point should be evident. Finally we will distinguish between three different cases as far as the distribution of the flow is concerned. Consider any given point $P_j$, $j \in I \cup II \cup III \cup IV$, i.e. $P_j$ is not on one of the axes, then we have:

- the continuous problem where the flow at $P_j$ is arbitrarily divided and assigned to the two directions, i.e. $x_j \in [0; 1]$
- the integer problem where the division at $P_j$ is integer, i.e. $x_j \in [0; 1]$ and $s_j x_j \in \mathbb{N}_0$
- the batch problem where the total supply at $P_j$ is sent either way, i.e. $x_j = 0 \vee 1$.

## 3. The Continuous Global Flow Problem

Consider the continuous problem first. Define new variables

$x_{IE} = \sum_{j \in I} s_j x_j$, $x_{IIW} = \sum_{j \in II} s_j x_j$, $x_{IIIW} = \sum_{j \in III} s_j x_j$, $x_{IVE} = \sum_{j \in IV} s_j x_j$

as well as $x_{IN} = s_I - x_{IE}$, $x_{IIN} = s_{II} - x_{IIW}$, $x_{IIIS} = s_{III} - x_{IIIW}$ and $x_{IVS} = s_{IV} - x_{IVE}$. Then the continuous global flow problem is

$$P_{CG}: \quad \text{Min } z = \max \{(f_N + x_{IN} + x_{IIN}), (f_W + x_{IIW} + x_{IIIW}),$$

$$(f_S + x_{IIIS} + x_{IVS}), (f_E + x_{IVE} + x_{IE})\}$$

$$\text{s.t.} \quad x_{IE} + x_{IN} = s_I$$

$$x_{IIN} + x_{IIW} = s_{II}$$

$$x_{IIIW} + x_{IIIS} = s_{III}$$

$$x_{IVS} + x_{IVE} = s_{IV}$$

$$x_{..} \geq 0 \quad \forall \, ..$$

Note that the continous global flow problem includes eight variables, four equality constraints and the usual nonnegativity constraints, no matter how many points are originally given. Hence the problem can be solved in constant time. A solution for $P_{CG}$ will indicate for any given experiment P* how many units in the first quadrant should first be shipped to the northern axis and then to P* and how many of those supply units are first to be directed to the eastern axis and then westward to P* (and similarly for the other three quadrants). Such a

solution does not answer the question at any point $P_j$ where its supply is to be directed. This problem is addressed later. Also note that this accumulation approach - at least from the viewpoint of a mathematical formulation - does not work with the integer or the batch problem since the nondivisibility has to be guaranteed for the supplies of the individual points. Even though the solution of $P_{CG}$ is simple, it is analyzed in the following. Observe that $P_{CG}$ can be transformed by the usual methods into a transportation problem with minisum objective and variable upper bounds $\kappa$. Such a transformation results in the following network.

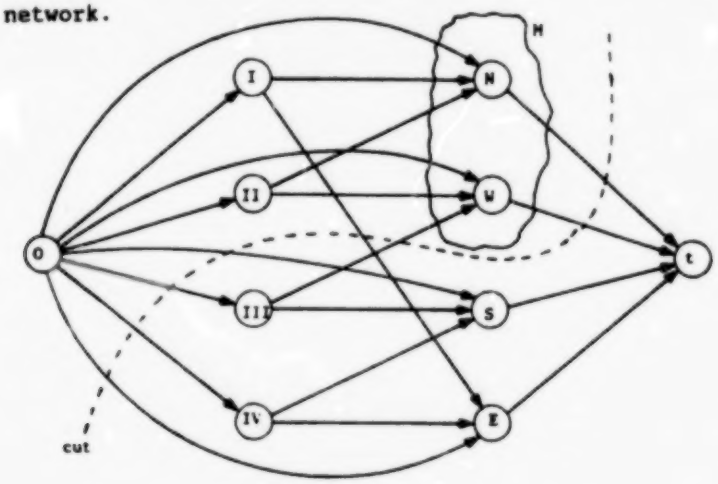


Figure 2

where the nodes I, II, III and IV represent the quadrants, N, W, S and E stand for the respective axes and O and t are a supersource and a supersink, respectively. The arc capacities are exemplarily displayed in figure 3.

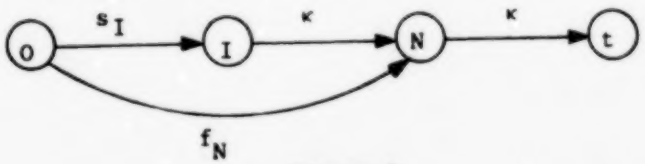


Figure 3

The connection between the network in figure 2 and the original problem in the plane is as follows. The flow in the arc (O, I) denotes the number of units within quadrant I (which equals $s_I$ in all feasible solutions), the flow in arc (I, N) determines the value of $x_{IN}$, the

$+ s_{IV} + f_E \overset{!}{\geq} s + f$ or $\kappa \geq \frac{1}{3}[s_{II} + s_{III} + f_N + f_W + f_S]$. In general, $\kappa \geq \kappa_3 := \frac{1}{3}$ max {supply (two adjacent quadrants plus their three adjoining axes)}.

Case 4: $|M| = 4$. Then $c(M) = 4\kappa \overset{!}{\geq} s + f$ or $\kappa \geq \kappa_4 := \frac{1}{4}[s + f]$ which, incidentally, assures that the flows in all $(0, N_1)$ and $(0, N_2)$ arcs are equal to their respective capacities.

Now $\kappa := \max\{\kappa_1, \kappa_2, \kappa_3, \kappa_4\}$ is the minimal capacity necessary to allow an (0, t) flow with value s + f. Note that $\kappa$ equals the objective function value z in the problem $P_{CI}$.

One possible way to proceed from here is as follows. First determine the minimal value of the variable capacity $\kappa$ as shown above. Then, with all capacities fixed, solve a regular flow problem in the graph of figure 2. Actually, it is even possible to do without solving the flow problem explicitly, its solution can easily be determined by means of the original parameters s. and f. for each of the above four cases directly. Due to space limitations the corresponding relations are omitted here. Note that in this problem - as in most minimax problems - there are many alternative optimal solutions allowing a hierarchy of objectives.

## 4. The Continuous Individual Flow Problem

In this section it is described how to obtain a solution to the individual problem from the solution of the global problem. The routing could, of course, be done almost arbitrarily as long as the values $x_{IN}$, $x_{IIN}$, ... are respected. It is, however, possible to easily find solutions that avoid some of the problems outlined in figure 1 in section 2. The suggested procedure is shown graphically for flows in the second quadrant. A ray with end point at the origin pointing west is turned clockwise. The supplies of all points covered by the ray in this movement are accumulated until the (known) value of $x_{IIW}$ is reached or exceeded. Then the supplies of all points which were covered in this fanning movement or sweep are shipped to the western axis whereas the supplies of all remaining points in the second quadrant are shipped to the northern axis. It may be (and it usually is) necessary to split

the supply at one of the points, namely where the value of $x_{IIW}$ was exceeded. This procedure is repeated for all quadrants resulting in a solution with the following properties: there are no crossings, degenerate or otherwise. Split supplies exist at no more than four points, at most one in each quadrant. Moreover, triple junctions may have been generated. Then each of the four axes has a catchment area (not unlike those in Voronoi diagrams) which can be visualized in figure 4.

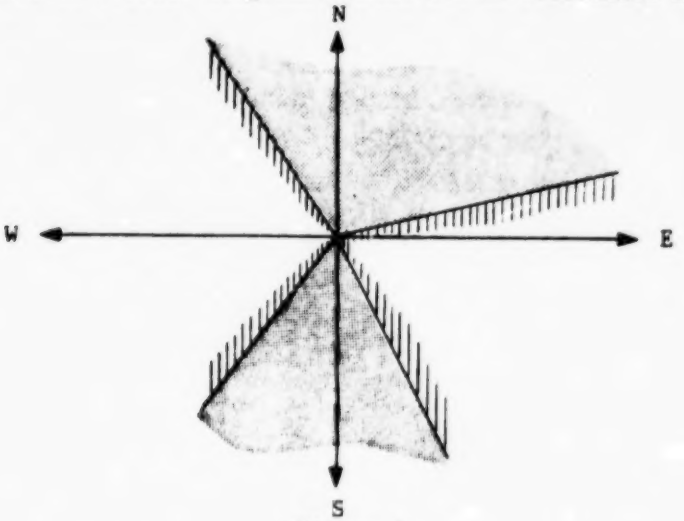


Figure 4

It is now possible to prove

Lemma 3: No point on a line segment or at a non-supply point can be the unique optimal minimax center.

Again the proof is easy and it is only sketched here. Let the current experiment be located on a line segment or at a non-supply point. If one moves the location of the experiment into one of the adjacent areas, then all previous solutions remain attainable and some additional solutions exist. In particular, the supplies of the points previously located on an axis can now be shipped on two, rather than on one route to P*. Hence the new "area solution" can never be worse but is possibly better that the line or non-supply point solution. This means that the optimal solution must be located at one of the n supply points or in one of the $O(n^2)$ areas. Note that both cases can be realized. Consider four points forming a rectangle with all corner points having equal supplies of, say, 1. The minimax center P* is located in the

rectangle and its objective function value $\bar{z} = 1$ whereas any point solution has an objective function value of $1\frac{1}{2}$. On the other hand, the triangle $P_1 = (1, 0)$, $P_2 = (0, 0)$ and $P_3 = (0, 1)$ with $s_1 = 2$, $s_2 = 1$ and $s_3 = 4$ has its minimax center at $P_3$ with $\bar{z} = 1\frac{1}{2}$ whereas no area solution has a better objective function value than 2.

Fortunately an exhaustive search of all $O(n_2)$ areas is not necessary. The following considerations may be helpful to cut down the computational burden substantially. First accumulate the supplies of the given points from left to right until the value of s/4 is reached or exceeded for the first time. At this $\chi$-coordinate a north-south barrier is constructed. This procedure is repeated from the right to the left resulting in another north-south barrier. The same procedure is used on the $\gamma$-coordinates from top to bottom resulting in an east-west barrier and finally from the bottom to the top, creating another east-west barrier. The four barriers form a rectangle. If the rectangle degenerates to a point, this point is optimal. If it degenerates to a bounded straight line consisting of some line segments as well as some given points, then one can find the $L_1$-median on that covering line. Either the $L_1$ median or one of its four adjacent areas is optimal.

Suppose now that the rectangle includes at least one area. Any experiment P* in an area within the rectangle generates quadrants I, II, III and IV, so that the combined supply of any two adjacent quadrants is strictly less than three quarters of the total supply. In other words, $\max \{\frac{1}{4}(s_I + s_{II}), \frac{1}{4}(s_{II} + s_{III}), \frac{1}{4}(s_{III} + s_{IV}), \frac{1}{4}(s_{IV} + s_I)\} < \frac{1}{4}(s + f) = \kappa_4$ and thus the optimal flow value for any P* within an area in the rectangle equals either $\kappa_2$ or the constant $\kappa_4$. Consider now one horizontal sequence of areas. Clearly, by moving an experiment P* from left to right the values of $s_I$ and $s_{IV}$ are monotonically nonincreasing whereas $s_{II}$ and $s_{III}$ are monotonically nondecreasing. Based on this unimodality it can be concluded that the upper envelope $\kappa = \max \{\kappa_2, \kappa_4\}$ is unimodal (downwards) within the limits of the rectangle. It is, however, not necessarily strictly unimodal prohibiting a direct application of a Fibonacci-type search method as in /SH 79/. Clearly, the minimal value of $\kappa$ for a point in an area in the sequence of areas is achieved where $\kappa = \kappa_2 = s_I$ or $s_{IV}$

to the left and $\kappa = \kappa_2 = s_{II}$ or $s_{III}$ to the right or where $\kappa = \kappa_4$. In the latter case the search procedure can be aborted since no point in any area can have a better solution. In the former case, the "critical point" could be found by a binary-type search with the rejection rules based on the quadrant with maximal supply but not on the value of the supply. There are $O(\log n)$ experiments necessary each requiring constant time once the values of $s_I, \ldots, s_{IV}$ are known. As a result, the best area solution can be found in $O(n \log n)$ time if a clever updating scheme for $s_I, \ldots, s_{IV}$ is used.

For practical purposes the use of rejection rules such as the ones in /EI 84/ appears preferable. First the best supply point solution is determined. If its objective function value $\bar{z}_s$ is no larger than $\frac{1}{2}[s + f]$, it is optimal. Otherwise an experiment P* is located in any area which was not rejected previously. Consider the following four cases.

(i) $\kappa = \kappa_2 = s_I$, then no better area solution exists southwest of P* (and similar for $s_{II}$, $s_{III}$ and $s_{IV}$)

(ii) $\kappa = \kappa_3 = s_I + s_{II}$, then no better area solution exists south of P* (and similar for $s_{II} + s_{III}$, $s_{III} + s_{IV}$ and $s_{IV} + s_I$)

(iii) $\kappa = \kappa_4$, then no better area solution exists at all

(iv) $\kappa \geq \bar{z}_s$, then the previously determined optimal supply point solution is optimal.

Based on (i), (ii), (iii) and (iv), rejection rules can easily be formulated. These rules are quite effective, with them the optimal area solution is usually found within a few iterations.

## 5. The Combinatorial Problem and Computational Experience

In this section the batch problem with and without additional constraints is discussed. The batch individual flow problem $P_{BI}$ has the same objective as $P_{CI}$ but its constraints are $x_j = 0 \vee 1 \; \forall \; j$. First an important result concerning $P_{BI}$ is developed. Let the set $H = \{1, 2, \ldots, n\}$ and let $w_j \in \mathbb{N} \; \forall \; j \in H$ be positive weights. Then the problem of finding a subset $J \subseteq H$ which minimizes $\left| \sum_{j \in J} w_j - \sum_{j \in H \setminus J} w_j \right|$ is known to be NP-complete, see /GJ 79/ or /HU 82/. Using some elementary transformations, see for instance /EPS 85/ the problem can be

written as MIN z, s.t. $z \geq \sum_{j \in J} w_j - w/2$, $z \geq \sum_{j \in H \setminus J} w_j - w/2$ which, in turn, is equivalent to the minimax problem Min z = max $\{ \sum_{j \in J} w_j - w/2, \sum_{j \in H \setminus J} w_j - w/2 \}$. The selection of J does, however, not depend on the constant w/2, thus one may also determine the set J via Min z = max $\{ \sum_{j \in J} w_j, \sum_{j \in H \setminus J} w_j \}$. This problem could be solved by assigning arbitrary coordinates to the given elements in, say, the nonnegative quadrant. Then the origin is selected as an experiment and the batch problem $P_{BI}$ is solved. The resulting flows on the northern and eastern axes in the vicinity of the origin are $\sum_{j \in J} w_j$ and $\sum_{j \in H \setminus J} w_j$, respectively (or vice versa) and 0 on the southern and western axes. In other words, the NP-complete subset problem introduced at the beginning of this section was shown to be a special case of the batch problem $P_{BI}$. Thus one can state

Theorem 4: The problem $P_{BI}$ is NP-complete.

In the following a branch and bound procedure is described which finds an optimal flow pattern for the batch problem with the additional condition that crossings are prohibited based on a given location of P*. In particular, let P* be the location which is optimal for the continuous problem. Thus the proposed method starts with a solution of the continuous relaxation as described in the second section. The objective function value of the continuous problem provides a lower bound on the batch solution. As noted earlier, the sweep produces a solution with no more than four points where supplies are split. At each of these four points the total supply may be shipped into exactly one of two directions. This leaves sixteen distinct possibilities achieved by simple "rounding", each of which represents a crossing-free batch solution. The best one of these solutions provides the upper bound. Now we go back to the continuous relaxation. Any one of the split points, say $P_j \in I$, is selected and a binary branching is performed by fixing the flow from $P_j$ to the northern and the eastern axis, respectively. But whenever the flow of $P_j$ is fixed to one axis, flows of other points may have to be fixed as well since otherwise crossings

may result. This "fixing" step is repeated until all points with non-fixed supply directions are free to be directed to any of their two axes without crossing any of the routes already fixed. All supplies whose directions are fixed are thought of being located on the axis they are directed to and at this point the continuous relaxation is solved again. Its solution produces a new lower bound. Now the aforementioned sixteen cases are enumerated again, hoping that a better batch solution and thus an improved upper bound can be found quickly. These steps are repeated as long as necessary, i.e. until lower and upper bound coincide. Clearly, there is some degree of freedom in this method particularly at what node in the solution tree to branch and which directions to fix. As far as the selection of a branching node is concerned, the earliest generated node which is not an end node was chosen.

When fixing the directions of a flow, that point and direction was selected which resulted in the maximal number of additional directions that could be fixed. It was thought that this strategy would keep the tree from getting very deep. It turned out that this was only partially true. In particular, frequently those points which generated many additional fixings when directed to one axis generated few or no additional fixings when directed to the other axis. This caused the solution tree to become quite asymmetric, i.e. few levels on one and many levels on the other side.

A series of tests was designed in order to test the above procedure. Points were generated randomly on a 100 x 100 grid with integer coordinates, and integer supplies between one and twenty. The number of points was chosen between 15 and 48. In all, twenty problems were solved with our own PASCAL code on an IBM-PC microcomputer using a hard disk and a Turbo Pascal compiler. In order to evaluate the results, define $\tilde{\Delta}$ as the percentual deviation of the objective function value of the best batch solution achieved by "rounding" the continuous solution. Similarly, let $\bar{\Delta}$ denote the percentual deviation of the z-values of the optimal batch solution from the optimal continuous solution. Finally, $\Delta$DIR symbolizes the percentual number of directions of flows that were changed from the optimal continuous to the optimal batch solution. Then the results are:

$\tilde{\Delta} \in [3.76; 24.27]$ with an average of 8.45

$\bar{\Delta} \in [0; 7.87]$ with an average of 1.57

$\Delta DIR \in [12.12; 60]$ with an average of 24.4

In other words, "rounding" the continuous solution to a batch solution increases the objective function value by roughly 8½ percent, whereas the optimal batch solution has a z-value which is only about 1½ percent higher than the one of the continuous solution. However, roughly a quarter of the supplies are rerouted to a different axis. A probabilistic analysis yields a similar value for $\tilde{\Delta}$; it also explains why the optimal continuous solution was always located at a given supply point $P_i$ with $\kappa = \frac{1}{4}[s - s_i + f]$.

## 6. Conclusion

In this study we have exclusively focused upon minimizing chaos as expressed by the magnitude of the flow while avoiding crossings and - in the preceding section - ensuring simplicity of the transportation or evaucation plan - the splitting of units. The factor being neglected here is the friction generated by all types of junctions. Consider for instance a "double junction" where two flows meet. The sum of the two flows does not appear to measure the friction very well. Certainly flows of 500 and 500 units, respectively, generate much more friction than flows of 980 and 20 units even though the sums in both cases are equal. Possibly the product of the flows is a better measure for the amount of friction at a junction. In this context, another important area is the discussion and evaluation of the trade-off between minimax flows on the one hand (possibly with large friction factors) and minimal friction factors on the other hand (possibly coupled with large flows). Results of traffic studies may help to find a function that combines friction and magnitude of flow, so that congestion, delays and chaos can be minimized. Finally one has to consider the dynamics of the system. With the point P*, e.g. the emergency shelter being located, the problem becomes a pure routing problem whose solution depends solely on the supplies (of people) at the respective points (buildings). Clearly, the population in the various buildings in the city changes over time. Office buildings

are usually deserted after 5:00 p.m. and high-rise apartment buildings are most heavily populated in the evening, night and in the early morning. One way would be to use past observations to set up a "population function" indicating the number of people in a building based on the time of day, the day of the week and the week of the year (to account for seasonal influences). Another fully automated method would be to count the number of people entering and leaving the major buildings using tunrstiles, electronic eyes or similar machines.

References

/EI 84/ H.A. Eiselt: Minimax Locations in Trees, working paper #84-006, Faculty of Commerce and Administration, Concordia University, Montreal, 1984.

/EP 84/ H.A. Eiselt, G. Pederzoli: A Location Problem in Graphs, New Zealand Operational Research 12, 1984.

/EPS 85/ H.A. Eiselt, G. Pederzoli, C.-L. Sandblom: Operations Research: Continuous Optimization Models, W. de Gruyter, 1985.

/GJ 79/ M.R. Garey, D.S. Johnson: Computers and Intractability, W.H. Freeman & Co., 1979.

/HU 82/ T.C. Hu: Combinatorial Algorithms, Addison-Wesley, 1982.

/SH 79/ R.D. Shapiro: Fibonacci Search in A.G. Holzman (ed.): Operations Research Support Methodology, M. Dekker, 1979.

/SMP 82/ Y. Sheffi, H. Mahmassani, W.B. Powell: A Transportation Network Evacuation Model, Transportation Research 16A, 1982.